A SHORT INTRODUCTION TO LUMINOSITY FUNCTIONS OF GALAXIES

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This brief review is intended to provide students with an introduction to the concepts and terminology associated with galaxy luminosity functions, particularly as used in the discussion of clusters of galaxies. Several of the lecturers refer in passing to L^*, the RORF, and other simple concepts which may be unfamiliar to beginning graduate students. The purpose of this lecture is to clarify the idea of a luminosity function, and describe some commonly used forms for the optical, radio, and infra-red luminosity functions of galaxies.

1) Definition and Uses of the Luminosity Function

The luminosity function is a distribution function, specifically the distribution of luminosities of objects in a sample. Luminosity itself is difficult to measure, since the total or bolometric luminosity requires both accurate distance measurements and integration of each object's spectrum over all frequencies. Generally we measure specific luminosity, L_ν, over a given band, or range of frequency, so the units of L_ν are erg $\sec^{-1}$ Hz^{-1}. Then the luminosity function is $n(L_\nu)$, where $n(L_\nu)\, dL_\nu$ is the number of galaxies with luminosity in the range L_ν to $L_\nu + dL_\nu$; $n(L_\nu)$ has units Mpc^{-3} (watt $Hz^{-1})^{-1}$. The integral of $n(L_\nu)dL_\nu$ over all luminosities is just the density of galaxies, n.

Note that if $n(L_\nu)$ is described by a power law, for L to be finite requires that the power law index be less than -1 at high luminosities, and greater than -1 at low luminosities, where L is the total luminosity of all galaxies in a unit volume, i.e.,

$$L\ (\text{watt Hz}^{-1}\ \text{Mpc}^{-3}) = \int_0^\infty L_\nu\ n(L_\nu)\ dL_\nu$$

A simple combination of power laws is often a useful approximate form to assume for a luminosity function, i.e.,

$$1. \qquad n(L) = \begin{cases} n_1 L^{-\alpha_1} & (L < L_0) \quad (\alpha_1 < 1) \\ n_2 L^{-\alpha_2} & (L > L_0) \quad (\alpha_2 > 1) \end{cases}$$

What we know about luminosity functions generally comes from the distribution of fluxes in a survey which is complete in some way. The best is a volume-limited sample (meaning every galaxy in the volume has been measured). This is rarely available, since all surveys have some minimum detectable flux, S_{min}, which translates into a cutoff distance $r_{cut} = \sqrt{L/4\pi S_{min}}$ which is a function of luminosity. So the sample size is a function of luminosity. This bias can be corrected if we know that the distribution of galaxies is homogeneous, i.e., that the total density

$$n = \int_0^\infty n(L)\, dL$$

is independent of position. In general n is a strong function of position; it varies by several orders of magnitude between rich clusters and voids; indeed, the study of this function is the main subject of this course.

If the distribution of objects is homogeneous, so that n is a constant independent of position, which presumably olds when we average over very large scales, then we can easily evaluate some simple integrals of the luminosity function which apply to a flux limited sample. The number of objects brighter than the minimum flux S_{min} is just given by

$$2. \qquad N_>(S_{min}) = \int_0^\infty 4\pi r^2 dr \int_{4\pi r^2 S_{min}}^\infty n(L)\, dL$$

$$= \int_0^\infty dL\, n(L) \int_0^{R_{cut}(L,S_{min})} 4\pi r^2\, dr$$

where we have simply interchanged the integration over volume and over luminosity. (Note: the distance r, which on small scales is simply cz/H_o, generalizes in a Friedman universe with $\Lambda = 0$, $\Omega = 1$ to

$$r = \frac{c}{H_0}\frac{2}{1+z}\left[z - (\sqrt{1+z}-1)\right]$$

see Condon (1984a). In the simplest case of "standard candles" the luminosity function is

$$n(L) = n\ \delta(L-L_0)$$

so that

$$N_>(S_{min}) = \frac{4}{3}\pi n \left(\frac{L_0}{4\pi S_{min}}\right)^{\frac{3}{2}}$$

In magnitude notation $N_>(S_{min}) \propto S_{min}^{-3/2}$ becomes $N_<(m) \propto m^{0.6}$. The "differential source count" function, $n(S_{min}) = -dN_>(S_{min})/dS_{min}$ depends on S_{min} to the minus 5/2 power, and the total flux from all sources is proportional to $\int S\, n(S)dS$ which diverges. This is a statement of Olber's paradox, which can only be resolved in a Euclidian universe if the luminosity function evolves, i.e., changes with time. The only tracers for which we have complete flux limited samples reaching to large redshifts ($z > 1$) are QSO's and radio sources, both of which apparently have luminosity functions which evolve strongly on cosmological time scales.

In a flux limited sample the contribution to n(s) of sources of different luminosities is most easily seen in von Hoerner's (1973) "visibility function":

3. $$\phi(L) = L^{\frac{5}{2}}\ n(L)$$

whose dimensions are watt$^{3/2}$ Hz$^{-3/2}$ pc^{-3}, which is usually converted to Jy$^{3/2}$. A plot of log $\phi(L)$ vs. log L immediately shows what range of luminosities contribute most to a flux limited sample, since

4. $$n(S) = 4\pi S^{-\frac{5}{2}} \int_{-\infty}^{\infty} \phi(L)\ d\log L$$

(see Condon 1984a,b).

A useful integral of the luminosity function gives the median distance to objects in a flux limited sample, $r_{1/2}$, given by

$$5. \quad \int_0^{r_{\frac{1}{2}}} 4\pi r^2 dr \int_{4\pi r^2 S_{min}}^{\infty} n(L) dL = \int_{r_{\frac{1}{2}}}^{\infty} 4\pi r^2 dr \int_{4\pi r^2 S_{min}}^{\infty} n(L) dL$$

where again we can interchange integration to get

$$6. \quad \int_0^{L_{\frac{1}{2}}} n(L) \left(\frac{L}{4\pi S_{min}} \right)^{\frac{3}{2}} dL = \int_{L_{\frac{1}{2}}}^{\infty} n(L) \left(\frac{L}{4\pi S_{min}} \right)^{\frac{3}{2}} dL$$

where $L_{1/2} = 4\pi S_{min} r_{1/2}^2$. This can easily be evaluated for n(L) having the simple form of equation 1. For example, if $n_2 = 0$ and $\alpha_1 = 0$ [i.e., a step function n(L)], we find simply that $r_{1/2}$ is $0.64 \sqrt{L_o/4\pi S_{min}}$. Using $\phi(L)$ equation 6 becomes simply

$$\int_{-\infty}^{x} \phi(L) \, d\log L = \int_{x}^{\infty} \phi(L) \, d\log L$$

where $x = \log L_{1/2}$, which is the obvious median value of $\phi(L)$ when plotted vs. log L.

OPTICAL LUMINOSITY FUNCTIONS

If we have a sample with distance information for every object, so that we know $L(r,S) = 4\pi r^2 S$ for each galaxy, we can define a volume over which the sample is complete to any arbitrarily low luminosity, and compute the density of galaxies as a function of luminosity. This is the approach taken by Schechter (1976), using a sample from the Reference Catalog of Bright Galaxies (de Vaucouleurs and de Vaucouleurs 1967) which is magnitude limited at $B^o = 11.75$. An alternative method to derive n(L) is to study rich clusters which are distant enough that $1/r^2$ is roughly constant for all members and rich enough that confusion due to foreground and background galaxies in the field is not a major problem (Oemler 1974). A compendium of various functional forms for n(L) is given by Felten (1977). The form chosen by Press and Schechter (1974) is very popular, not because it is simple, but because it follows from a theoretical analysis of self-similar gravitational condensation in the early universe. This is a three parameter function of the form:

$$7. \qquad n(L)\,dL = n\left(\frac{L}{L^*}\right)^{\alpha} e^{-\frac{L}{L^*}}\, d\left(\frac{L}{L^*}\right)$$

where the parameters are n (the total density), α (the power law slope for very low L), and L^* (the luminosity of the "break" where the slope of n(L) changes rapidly). For L greater than L^*, n(L) decreases exponentially. Typical values for α lie in the range $-1.5 < \alpha < -1$, so integrating over all luminosities the total number of galaxies diverges. This is not necessarily unphysical, since the total luminosity, $\int L\, n(L)dL$ remains finite. Integrals of the Schechter function can often be expressed in terms of the incomplete gamma function (Davis 1964), e.g.,

$$8. \qquad \int_{L_0}^{\infty} n(L)\,dL = n\,\Gamma\left(\alpha+1, \frac{L_0}{L^*}\right)$$

An analog to the median luminosity galaxy is the "half-light" object, for which half the total luminosity of the sample comes from galaxies of higher luminosity, half from lower:

$$9. \qquad \int_{0}^{L_{\frac{1}{2}}} L\, n(L)dL = \int_{L_{\frac{1}{2}}}^{\infty} L\, n(L)dL$$

$$\int_{0}^{x_{\frac{1}{2}}} x^{\alpha}\, e^{-x}\, dx = \int_{x_{\frac{1}{2}}}^{\infty} x^{\alpha}\, e^{-x}\, dx$$

which gives $L_{1/2} = 0.16\ L^*$, i.e., about 2 magnitudes fainter than L^*. Expressing the luminosity parameter as a magnitude

$$M^* = M_\odot - 2.5 \log \left(\frac{L^*}{L_\odot} \right) \tag{10}$$

we find M^* typically has a value in the range $-23 < M^* + 5 \log h_{50} < -20$ in the blue.

If the parameters α and L^* of the Schechter function were independent of position (i.e., a global form for the luminosity function varying only in its normalization, n), then many problems could be greatly simplified. For example, distances to clusters could be estimated by measuring the number of galaxies vs. apparent magnitude, and fitting L^*. (A more sophisticated treatment of this idea is given by Schechter and Press, 1976.) The variation of the luminosity function among clusters has been studied by Dressler (1978) and more recently by Lugger (1986). Apparently there is variation of almost two magnitudes in M^* from cluster to cluster, although this variation is not clearly associated with cluster properties; in particular the presence of a cD does not imply a depletion of moderate luminosity galaxies, supporting Merritt's (1985) conclusion that cD's are not growing significantly by cannibalism at present. It remains an open question whether either the parameters L^* and α, or the form of the luminosity function itself, vary with large scale environment, e.g., between clusters and voids. This would be expected in a "biased galaxy formation" scenario (e.g., Dekel and Silk 1986) where the initial mass function of galaxies is strongly influenced by the local mass density.

Another question about the optical luminosity function which is currently discussed is whether different luminosity functions should be used for different galaxy types (Sandage et al. 1985, Binggeli 1987). If dwarf ellipticals and ordinary ellipticals are considered separately, the luminosity function of the larger galaxies is typically Gaussian; the same is true for irregulars and spirals. Virgo is the only cluster for which we have complete catalogs of galaxies faint enough to measure various luminosity functions for different galaxy types. It is known in many clusters that the relative abundance of different galaxy types is a strong function of the local density of galaxies (Dressler 1980), so it is not implausible that at least ellipticals and spirals might have different luminosity functions. This is certainly true at the very high end, where the (exclusively elliptical) cD's are often so far above L^* that their abundance is not well fit by the exponential cutoff of the Schechter function at the high end.

Assuming that different galaxy types have distinct luminosity functions, morphological segregation (e.g., Giovanelli and Haynes 1985, Dressler 1980) appears to require that the overall galaxy luminosity function vary with position, since varying the relative fractions of spiral, elliptical and S0 galaxies varies the contribution of each of their luminosity functions to the aggregate luminosity function. Since the colors of galaxies also correlate with type this means that the aggregate Schechter function parameters must also vary with color. Large scale structure in the galaxy distribution entails more than a variation in n^* with position; in fact we may need a multivariate luminosity function, for example $n(L,\rho)$ where ρ is the density of galaxies in the immediate environment. Data which could be used to derive this function are presented by Haynes in this volume.

RADIO LUMINOSITY FUNCTION

Flux limited radio source catalogs are dominated by elliptical galaxies with high luminosities [$\phi(L)$ peaks at $L \sim 10^{25.5}$ W Hz^{-1}] at very large distances (median $z \sim 1$). This is because the present luminosity function is quite flat; it is roughly fit by two power laws:

$$11.\quad L\,n(L) \cong \begin{cases} 10^{-4}\left(\dfrac{L}{10^{21}}\right)^{-0.44}\ \mathrm{Mpc}^{-3} & (10^{21} \lesssim L \lesssim 10^{24.75}) \\ 10^{-6}\left(\dfrac{L}{10^{25}}\right)^{-1.5}\ \mathrm{Mpc}^{-3} & (10^{24.75} \lesssim L \lesssim 10^{26.25}) \end{cases}$$

Above $L \cong 2\times10^{26}$ W Hz^{-1} it drops off. Evolution of the luminosity function is critical in determining the observed n(s), as discussed by Condon (1984a). Spiral galaxies make up a small fraction (~1%) of the radio sources brighter than 1 mJy. Typical spirals detected at this level are nearby, because for these $\phi(L)$ peaks near 10^{21} W Hz^{-1}. The luminosity function for spirals is given by (Condon 1984b):

$$12.\quad L\,n(L) \cong 0.1\left(\frac{L}{10^{19}}\right)^{-0.62}\ \mathrm{Mpc}^{-3} \quad (10^{19}<L<10^{21.5})$$

Above $L \cong 10^{21.5}$ watt Hz^{-1} the density of spirals drops off, and ellipticals dominate. An alternative form suggested by Hummel (1981, cf. Gavazzi and Jaffe 1986) is a log-normal luminosity function:

$$L\, n(L) \;\alpha\; \exp\left[\frac{\left(\log L - \log L_0\right)^2}{2\,\sigma^2}\right]$$

with $\sigma = 0.67$. L_o, the mean luminosity, is roughly proportional to optical luminosity, with $L_o \cong 10^{21}$ watt Hz^{-1} for $M_p \cong -20$.

To study the radio luminosity function of normal galaxies in the present epoch requires that we sift through a large sample of radio sources, selecting those few associated with optically bright, nearby galaxies. This preselection by optical properties causes a bias for optically brighter objects. To properly include this selection in a statistical treatment requires computation of the bivariate radio luminosity function (BRLF), f(P,M), which gives the fraction of all galaxies with optical magnitudes M to M+dM which have radio luminosity P to P+dP (Auriemma et al. 1977, Hummel et al. 1983). A simpler approach is to compute the radio-optical ratio function, RORF, given by f(R), the fraction of galaxies with radio-optical luminosity ratio in the range R to R+dR, where R is commonly defined as

13. $$R = S_{1400}\; 10^{\left(\frac{M_p - 12.5}{2.5}\right)}$$

(Condon 1980). Either of these functions can be written as a differential function, f(R), or integral (cumulative) function $F_>(R)$.

Results for f(R) are summarized by Gavazzi and Jaffe (1986), who find a log-normal distribution for f(R), with mean value $R_o = 10$ for Sc's and $R_o = 25$ for Sb's, and width $\sigma = 0.67$ in log R, i.e., a factor of five in R.

INFRARED LUMINOSITY FUNCTION

The luminosity function in the far infra-red has been derived from IRAS data by Hacking and Houck (1987). The visibility function and the possibility of evolution in this luminosity function are discussed by Hacking, Condon and Houck (1987).

Ratio functions of radio-infrared luminosity and radio-optical luminosity are presented by Hummel et al. (1988). The far IR-radio luminosity ratio is remarkably constant among spiral galaxies (Helou et al. 1985, DeJong et al. 1985, Beck and Golla 1988). The dispersion in this ratio is only about 0.2 (Wunderlich and Klein 1988), which means that f(R) is almost a delta function (although Seyferts and blue compact dwarfs show higher values than normal spirals). Thus the radio luminosity is a good predictor of the FIR luminosity for spirals.

Both the radio and FIR luminosity functions show a variation between cluster and field, which is most easily seen in the ratio functions (Gavazzi and Jaffe 1986, Bicay and Giovanelli 1987). The effect is that spiral galaxies in rich clusters which show evidence for HI deficiency tend to have lower radio continuum luminosity and cooler FIR colors ($S_{60\mu m}/S_{100\mu m}$) than similar field galaxies. This is often explained as a quenching of the star formation rate by stripping of the galaxy's interstellar gas. On the other hand, a few spirals in rich clusters show significantly enhanced radio emission, which suggests a burst of star formation during the first passage of a spiral through the intra cluster medium.

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